A Bayesian approach for parameter identification in elastoplasticity

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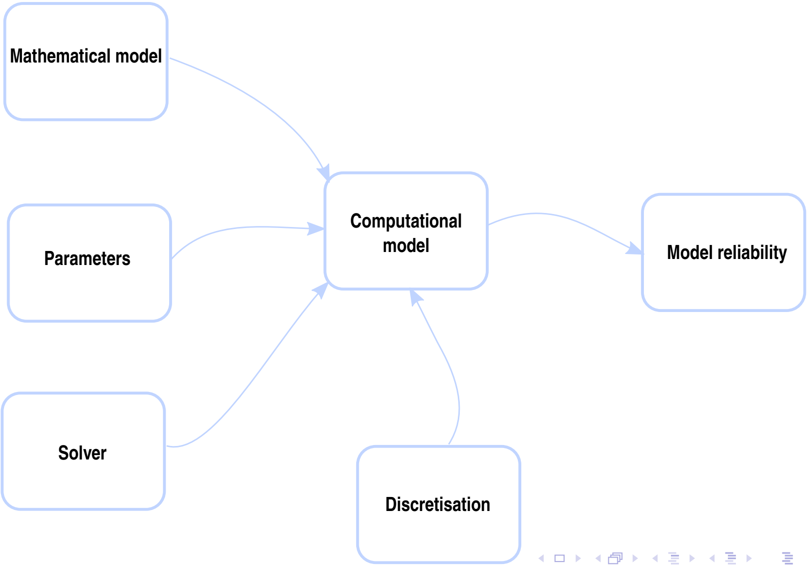
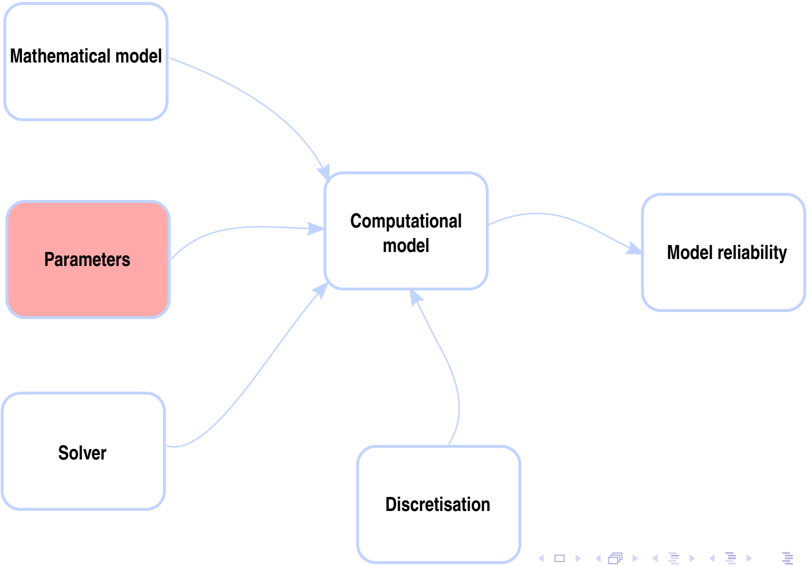
June 9, 2016

Introduction



Introduction



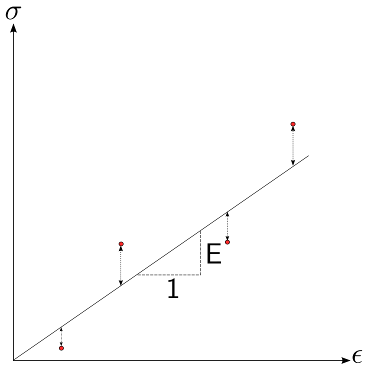
Error minimisation

Least squares method
(conventional approach):

$$\sigma = E\epsilon$$

$$J = \frac{1}{2}\sum_{i=1}^{N}(\sigma_i - E\epsilon_i)^2$$

$$\overline{E} = \underset{E}{\operatorname{argmin}}\, J(E)$$



Frequentist inference

Frequentist inference

10

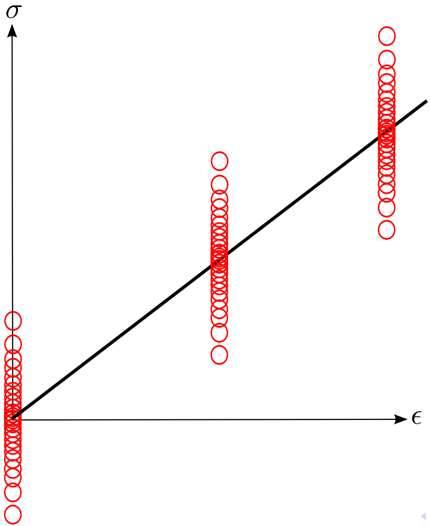
6

Frequentist inference

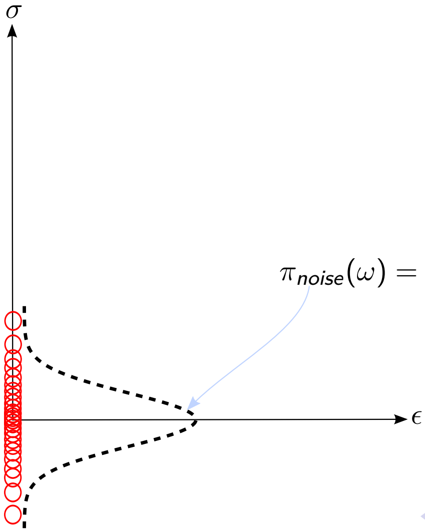
$Pr(head) = \frac{10}{16}$

$Pr(tail) = \frac{6}{16}$

Frequentist inference: Young's modulus identification



Frequentist inference: Young's modulus identification

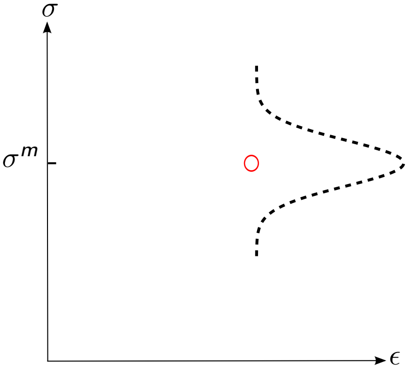


$$\pi_{noise}(\omega) = \frac{1}{\sqrt{2\pi} S_{noise}} \exp\Big(- \frac{\omega^2}{2 S_{noise}^2} \Big)$$

Frequentist inference: Young's modulus identification

$$\sigma^m = E\epsilon + \Omega$$

$$\Omega \sim \pi_{noise}(\omega)$$



Frequentist inference: Young's modulus identification

Method of maximum likelihood (ML):

$$\boxed{\pi(\sigma^m|E) = \frac{1}{\sqrt{2\pi}S_{noise}}\exp\Big(-\frac{(\sigma^m - E\epsilon)^2}{2S_{noise}^2}\Big)}$$

Frequentist inference: Young's modulus identification

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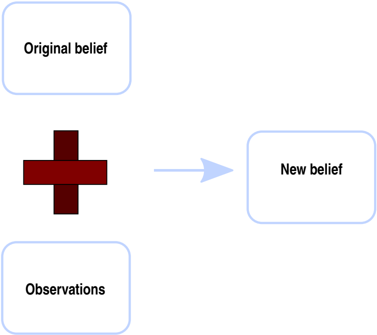
and for M measurments:

$$\boxed{\pi(\boldsymbol{\sigma}^m|E) = \frac{1}{(2\pi S_{noise}^2)^{\frac{M}{2}}}\exp\Big(-\frac{\sum_{i=1}^{M}(\sigma_i^m - E\epsilon_i)^2}{2S_{noise}^2}\Big)}$$

$$\boldsymbol{\sigma}^m = \Big[\sigma_1, \sigma_2, \cdots, \sigma_M\Big]$$

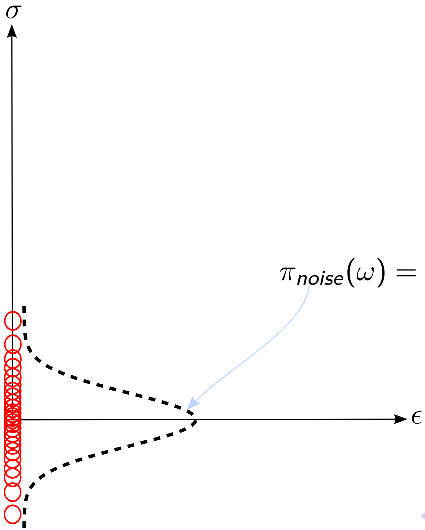
Bayesian inference

Bayesian inference



$$\pi(cause|effect) = \frac{\overbrace{\pi(cause)}^{prior} \times \overbrace{\pi(effect|cause)}^{likelihood}}{\underbrace{\pi(effect)}_{evidence}}$$

Bayesian inference: Young's modulus identification



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Bayesian inference: Young's modulus identification

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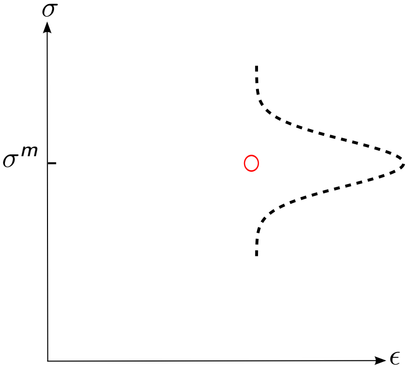

Bayesian inference: Young's modulus identification

Bayes' formula:

$$\pi(E|\sigma^m) = \frac{\pi(E)\pi(\sigma^m|E)}{\pi(\sigma^m)}$$

Bayesian inference: Young's modulus identification

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Bayesian inference: Young's modulus identification

Bayes' formula:

$$\pi(E|\sigma^m) = \frac{\pi(E)\pi(\sigma^m|E)}{\pi(\sigma^m)} \implies \frac{\pi(E)\pi(\sigma^m|E)}{C}$$

$$\boxed{\pi(E|\sigma^m) \propto \pi(E)\pi(\sigma^m|E)}$$

Bayesian inference: Young's modulus identification

$$\boxed{\pi(E|\sigma^m) \propto \exp\Big(-\frac{(E-\overline{E})^2}{2S_E^2}\Big)\exp\Big(-\frac{(\sigma^m - E\epsilon)^2}{2S_{noise}^2}\Big)}$$

Bayesian inference: Young's modulus identification

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and for M measurements:

$$\boxed{\pi(E|\boldsymbol{\sigma}^m) \propto \prod_{i=1}^{M} \exp\Big(-\frac{(E-\overline{E})^2}{2S_E^2}\Big)\exp\Big(-\frac{(\sigma_i^m - E\epsilon_i)^2}{2S_{noise}^2}\Big)}$$

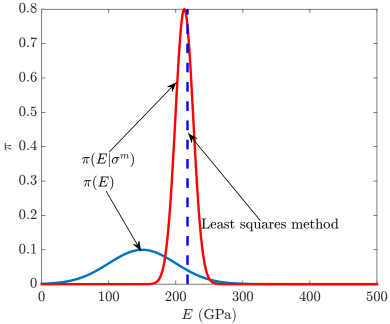
$$\boldsymbol{\sigma}^m = \Big[\sigma_1, \sigma_2, \cdots, \sigma_M\Big]$$

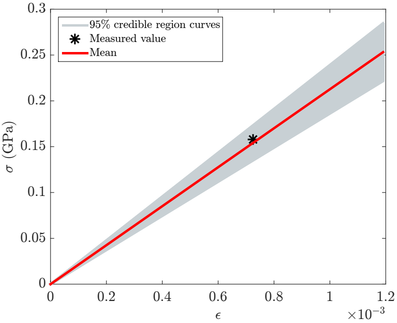
Bayesian inference: Young's modulus identification

$$\boxed{\pi(E|\boldsymbol{\sigma}^m) \propto \exp\Big(-\frac{(E-\mu)^2}{2S_{post}^2}\Big)}$$

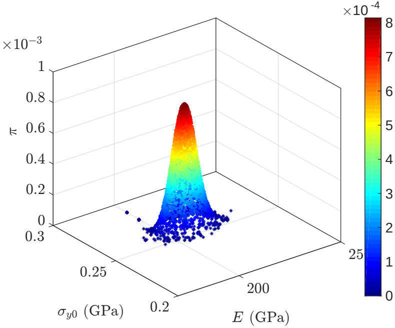
with $\mu = f(\sigma_i, \overline{E}, S_{nosie}, S_E, \epsilon_i)$
$S_{post} = f(\sigma_i, \overline{E}, S_{nosie}, S_E, \epsilon_i)$

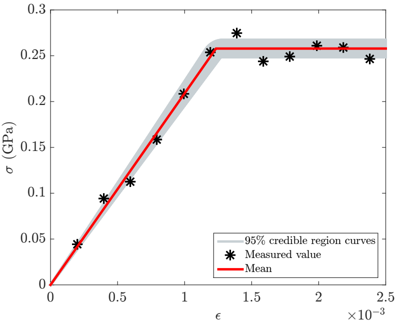
Bayesian inference: Young's modulus identification



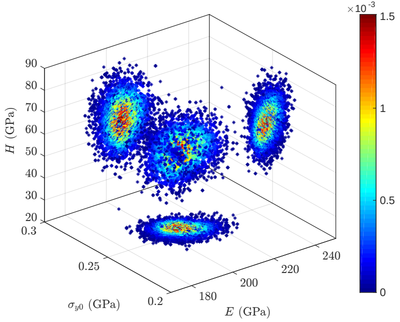


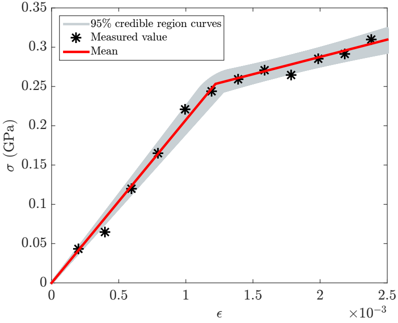
Bayesian inference: linear elastic-perfectly plastic model identification



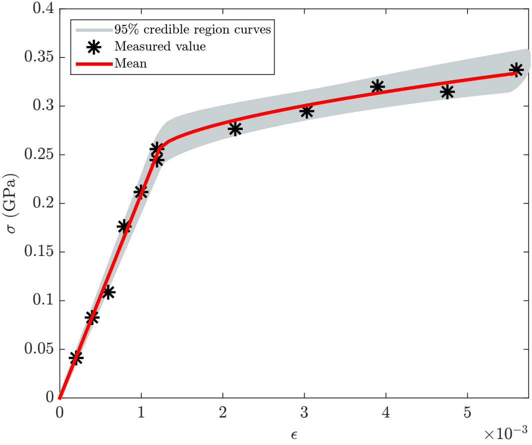


Bayesian inference: linear elastic-linear hardening model identification





Bayesian inference: linear elastic-nonlinear hardening model identification

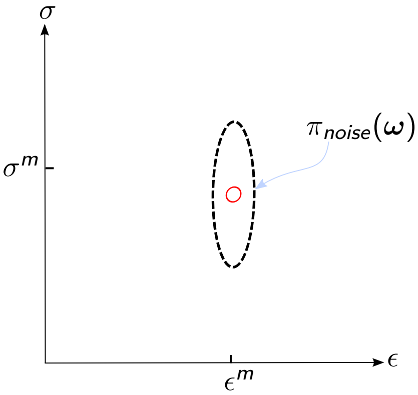


Bayesian inference: Young's modulus identification with double uncertainty

$$\sigma^m = E\epsilon + \Omega_\sigma$$

$$\epsilon^m = \epsilon + \Omega_\epsilon$$

$$\Omega_\sigma \text{ and } \Omega_\epsilon \sim \pi_{noise}(\omega)$$



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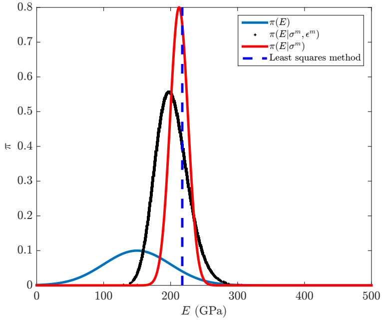
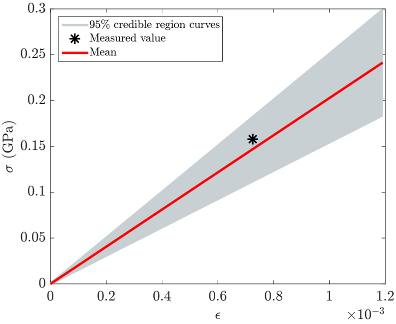
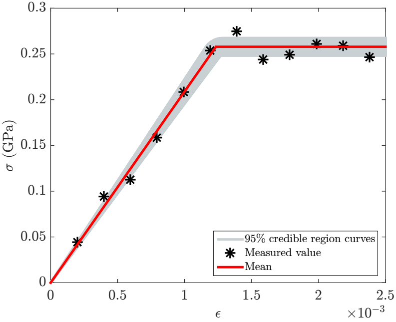
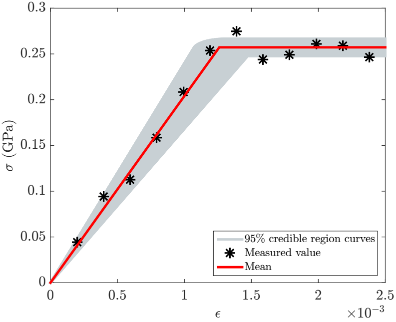




Bayesian inference: linear elastic-perfectly plastic with double uncertainty

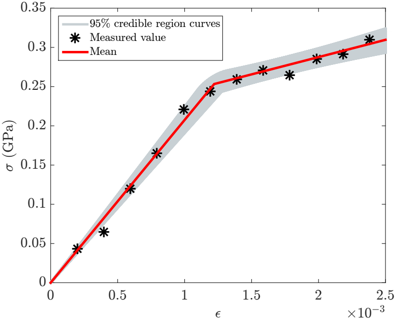


single uncertainty

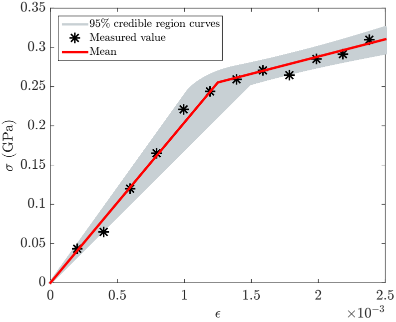


double uncertainty

Bayesian inference: linear elastic-linear hardening with double uncertainty

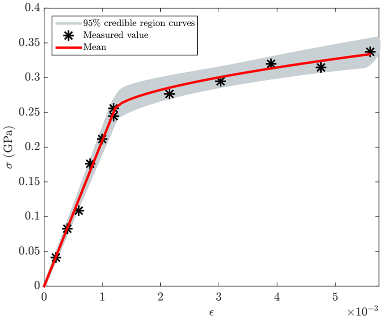


single uncertainty

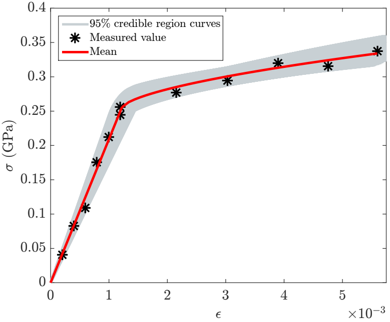


double uncertainty

Bayesian inference: linear elastic-nonlinear hardening with double uncertainty



single uncertainty



double uncertainty

Conclousion

- 'Closed form expression' of the posterior for:
  - linear elasticity,
  - elastoplasticity with perfect plasticity,
  - elastoplasticity with linear hardening, and
  - elastoplasticity with nonlinear hardening.

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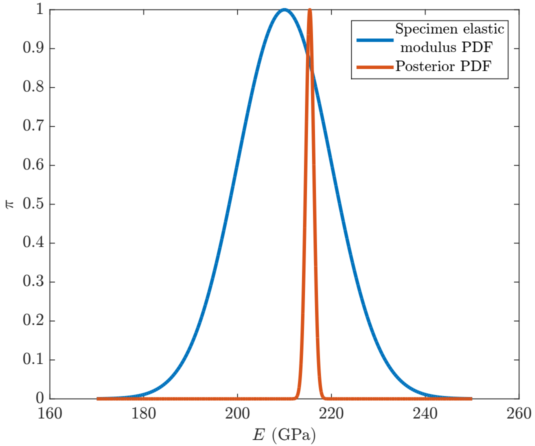
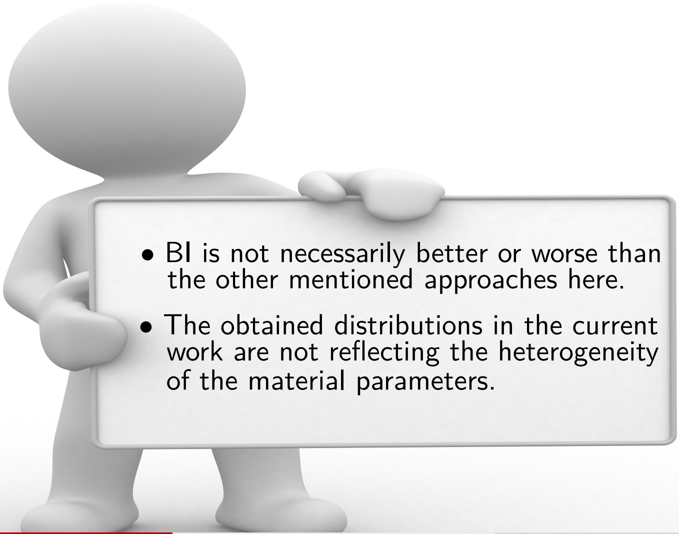
- The results of BI cannot directly be compared to those of the least squares method.
- The selected prior distribution determines if the results are better or worse than those of the least squares method.
- BI leads to a distribution for the considered parameters, however the resulting distribution do *not* reflect the heterogeneity of the material parameters.

Future work



Specimen elastic modulus PDF
Posterior PDF
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
π
160
180
200
220
240
260
E (GPa)

Take home message



A special acknowledgement

Computational Science Priority, University of Luxembourg

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